Limited Wavelength Translation in All-Optical WDM Mesh Networks

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Abstract— **We analyze limited wavelength translation in all-optical, wavelength division multiplexed (WDM) wrap-around mesh networks, where upto W wavelengths, each of which can carry one circuit, are multiplexed onto a network link. All-optical wavelength translators with a limited translation range permit an incoming wavelength to be switched only to a small subset of the outgoing wavelengths. Although more restrictive than full wavelength translation (which permits an incoming wavelength to be switched to any outgoing wavelength), limited wavelength translation is a topic of recent study, since current practical wavelength translators are capable only of limited translation. We consider the case where an incoming wavelength can be switched to one of k ($k = 2, 3$) outgoing wavelengths (called the *feasible wavelength set*), and we obtain the probability that a session arriving at a node at a random time successfully establishes a connection from its source node to its destination node. Our analysis captures the state of a feasible wavelength set at a network node, which allows us to obtain the probability of successfully establishing the circuit. Based on this probability, we quantify the benefits of limited wavelength translation by demonstrating that in mesh networks, it can obtain most of the performance advantages of full translation at a fraction of the cost. Our work is the first to analyze limited wavelength translation for mesh networks under a probabilistic model, and accurately predicts the network performance over a wider range of network loads than previous works.**

I. Introduction

Recent interest in all-optical networks ([22], [4], [6]) has focused attention on wavelength division multiplexing (WDM) (see, for instance, [8], [9], [10]) as a promising technique for utilizing in a natural way the terahertz bandwidth of optical fiber. By dividing the total available bandwidth into a number of narrower channels, each corresponding to a different optical wavelength, WDM provides the potential to fully exploit the enormous fiber bandwidth. A critical functionality for the scalability and improved performance of multihop WDM networks is *wavelength translation* [1], that is, the ability of network nodes to switch data from an incoming wavelength ϕ_i to an outgoing wavelength $\phi_j, j \neq i$. Two different classes of wavelength-routing nodes are important in this context: nodes with a *full-wavelength translation* capability, which translate an incoming wavelength to any outgoing wavelength and nodes with *no-wavelength translation* capability, which map each incoming wavelength to the same outgoing wavelength, the so called wavelength-continuity constraint (see, for instance, [3], [21], [14], [31], [5]). The requirement of wavelength continuity increases the probability of call blocking and can be avoided by the use of wavelength translation, as illustrated in Fig. 1.

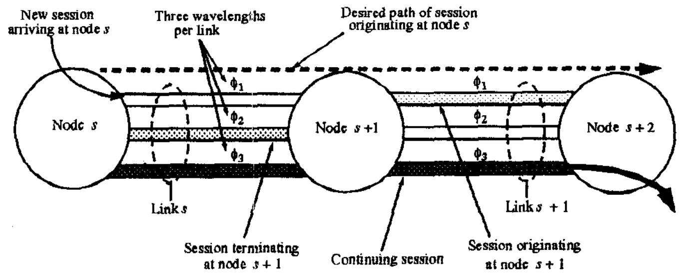

Fig. 1. The benefits obtained through wavelength translation are illustrated. The new session that arrives at node s requires a path that passes through node $s+2$, as shown. If the wavelength continuity constraint is imposed, the session will be blocked, since wavelength ϕ_1, which is the only free wavelength on link s is being used by an originating session on link $s+1$. If, however, wavelength translation is allowed, the session can be connected through a path that uses wavelength ϕ_1 on link s and ϕ_2 on link $s+1$. This reduces blocking and increases the number of sessions that can be served.

Although full-wavelength translation is desirable because it substantially decreases blocking probability (see [2], [16], [7], and [19]), it is difficult to implement in practice due to technological limitations. Since all-optical converters are still being prototyped in laboratories and are likely to remain expensive, researchers have turned their attention to searching for suitable alternatives [28]. A recent analysis by Subramaniam et al [24], found that there is no significant degradation in network performance even when only a few (as opposed to all) of the network nodes have a full-wavelength translation (conversion) capability. (A variant of this problem was first studied by Lee and Li [18], with a limited number of wavelength convertors per node.) A natural question that arises is, whether or not similar performance advantages can be obtained by using switches that have only a *limited wavelength translation* capability, where an incoming wavelength can be translated to only a small subset (as opposed to all) of the available outgoing wavelengths. This problem assumes practical significance when one considers that all-optical wavelength translators demonstrated in laboratories to date are, in general, capable only of limited translation (see, for example [13], [33], and [28]).

Realizing this limitation, researchers have recently begun studying limited wavelength translation in a systematic way in order to quantify its advantages vis-a-vis no wavelength translation and full wavelength translation. Yates et al [28] were the first to present a simple, approximate probabilistic analysis of this problem, while Gerstel et al [12], [11], were the first to examine limited wavelength translation for ring networks under a non-probabilistic model.

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Recently, Wauters and Demeester [32] have provided new upper bounds on the wavelength requirements for a WDM network under a static model of the network load, while Ramaswami and Sasaki [23] have provided a non-probabilistic analysis of the problem for tree networks and, under certain restrictions, for networks of arbitrary topology. Although their models are valuable and make no assumptions about traffic behavior, they only consider the static (one-time) problem, where there is a set of source-destination pairs that have to be connected and the objective is to serve as many such connection requests as possible. Such a model does not capture the dynamic nature of a network, where requests arrive and leave at random instants of time continuously over an infinite time horizon.

In this paper, we analyze limited wavelength translation in all-optical WDM mesh networks, where an incoming wavelength can be translated only to a subset consisting of k of the W outgoing wavelengths (i.e., to $k-1$ wavelengths in addition to itself). We call this *k-adjacent wavelength switching* and we call the set of output wavelengths the *feasible wavelength set*. Our aim is to address the following question: For a given network topology, how much of the performance improvement provided by full-wavelength translation over no wavelength translation can we achieve by using switches that have only a limited translation capability?

Our analysis for the wraparound mesh demonstrates that k-adjacent wavelength switching with only $k = 2, 3$ suffices to give performance significantly superior to that obtained with no wavelength translation, and close to that of full wavelength translation. The results that we obtain are very close to the corresponding simulation results, and predict network performance over a wider range of network loads than previous works. Furthermore, we find that the extent of translation k does not have to be a function of the total number of wavelengths W, as argued in previous works. This is because the models used by previous works are based either on trying multiple options simultaneously, or on using backtracking when establishing the connection, so that the improvements obtained in those cases increased both with k and with W. When a more realistic scenario is considered, where a setup packet examines at each hop only the outgoing wavelengths at that hop, instead of examining all possible paths from that hop onward, it turns out (see Sections II and III) that k is no longer a function of W. Thus, as long as $W \geq k$, the improvements obtained by limited wavelength translation are essentially a function only of the extent k of translation, and our results show that relatively small values of k yield performance close to that of full wavelength translation.

The remainder of the paper is organized as follows. We focus first on the 2-dimensional wraparound mesh as a representative network and analyze it in detail in Section II. In particular, in Subsection II-A, we explain the node model used, while in Subsections II-B and II-C, we discuss the auxiliary queueing system that we use for the case of $k = 2$-adjacent wavelength switching. In Subection II-D, we derive the expression for the probability that a session arriving at a random time successfully establishes a circuit. In Section III, we compare our analytic results for the success probability with those obtained by simulations for the two-dimensional mesh, and discuss our results. Our conclusions appear in Section IV.

II. Limited Wavelength Translation in the Torus Network

In this section, we examine limited wavelength translation in a 2-dimensional wraparound mesh network, more popularly known as the *torus* network. We focus on the mesh because it is a natural topology that fits the geography and has nodes of small degree, and we expect the results obtained for the mesh to be representative of the performance obtainable from a real network.

A $p \times p$ wraparound mesh consists of p^2 processors arranged along the points of a two-dimensional space with integer coordinates, with p processors along each dimension. Two processors (x_2, x_1) and (y_2, y_1) are connected by a (bi-directional) link if and only if for some $i = 1, 2$ we have $|x_i - y_i| = 1$ and $x_j = y_j$ for $j \neq i$. In addition to these links, wraparound links connecting node $(x_2, 1)$ with node (x_2, p), and node $(1, x_1)$ with node (p, x_1) are also present. The *routing tag* of a session with source node $x = (x_2, x_1)$ and destination node $y = (y_2, y_1)$, is defined as (t_2, t_1), where

$$t_j = \begin{cases} y_j - x_j & \text{if } |y_j - x_j| \leq \lfloor \frac{p}{2} \rfloor; \\ y_j - x_j - p \cdot \text{sgn}(y_j - x_j) & \text{if } |y_j - x_j| > \lfloor \frac{p}{2} \rfloor, \end{cases}$$

for all $j \in \{1, 2\}$, and $\text{sgn}(x)$ is the signum function, which is equal to $+1$ if $x \geq 0$, and equal to -1, otherwise.

In the analysis that follows, we consider the case of 2-*adjacent wavelength switching*, where each wavelength ϕ_j, $j = 0, 1, \ldots, W-1$ on an incoming link can be switched on the outgoing link, either to the same wavelength ϕ_j or to the next higher wavelength ϕ_{j+1}. (In [25], we have shown how our analysis can be extended in a straight-forward manner to the case of 3-adjacent wavelength switching.) We call the set $\{\phi_{j+1}, \phi_j\}$ of output wavelengths, the *feasible wavelength set* of input wavelength ϕ_j. For symmetry, we assume that the boundary wavelength ϕ_{W-1} can be switched to wavelengths ϕ_{W-1} and ϕ_0. Note that this is not merely an analytical convenience, but is also essential for performance to distribute the load equally among all wavelengths on a link.

A. The Node Model

In our model, connection requests or sessions arrive at each node of the torus over an infinite time horizon according to a Poisson process of rate ν, and their destinations are distributed uniformly over all nodes, except for the source node. Each session wishes to establish a circuit (or connection) to its destination node for a duration equal its holding time X, which is exponentially distributed with mean $\overline{X}$, and does so by transmitting a setup packet along a path to its destination. Setup packets use dimension-order routing to establish the circuit, that is, they either traverse all links along dimension 1 (horizontal) followed by all links along dimension 2 (vertical) or visa versa. In our model, links are bidirectional, so all incident links of a node (and their wavelengths) can be used simultaneously for transmission and reception. Our routing scheme is oblivious, or non-adaptive; that is, the path followed by the session is chosen at the source, and the setup packet insists on that path as it progresses from its source to its destination. If the setup packet of a session is successful in establishing the circuit, the wavelengths required by the session are reserved for the session duration. Otherwise, the session is reinserted randomly into an input queue for the node (which is ordered as per the times at which the sessions in it must retry), and tries again when its turn arrives. Thus, a session may try several times before it is finally successful. Sessions that are blocked during a particular trial are reinserted into the input stream and are eventually served. (The interarrival time between sessions inserted in the input queue is several times the interarrival time between external arrivals, so that the combined arrival process of the new sessions and the reinserted sessions can still be approximated as a Poisson process.) By contrast, in the call dropping model used in other analyses, sessions with longer path lengths are dropped with a higher probability. The call dropping model, in addition to treating unfairly long connections, also tends to overestimate the maximum throughput achievable, by favoring connections that require a smaller number of links.

In 2-adjacent wavelength switching, the switching of a new session arriving on a wavelength ϕ_j on an incoming link of a node (or a new session originating at the node that wishes to use wavelength ϕ_j on its first outgoing link), depends only on the availability of wavelengths ϕ_j and ϕ_{j+1} on the outgoing link. Because of symmetry, we focus, without loss of generality, on an incoming session that arrives over a particular wavelength, say ϕ_0, and wants to use outgoing link L of a node. Such a session can only be switched to wavelengths ϕ_0 or ϕ_1 of link L. We also consider the (wavelength, incoming link) pairs that can be switched to either wavelength ϕ_0 or ϕ_1 of link L (see Fig. 2). As shown in Fig. 2, D_1^1, D_1^2, and D_1^3 denote wavelength ϕ_1 on the incoming links D^1, D^2, and D^3, respectively, which can be switched only to wavelength ϕ_1 on link L. Similarly, D_{-1}^1, D_{-1}^2, and D_{-1}^3 denote wavelength ϕ_{-1} on links D^1, D^2, and D^3, respectively, which can be switched only to wavelength ϕ_0 on link L. (In our notation, wavelength ϕ_{-1} corresponds to wavelength ϕ_{W-1}.) Finally, D_0^1, D_0^2, and D_0^3 denote wavelength ϕ_0 on the incoming links D^1, D^2, and D^3, respectively, which can be switched either to wavelength ϕ_0 or to wavelength ϕ_1 on link L. We define the joint state of a pair of outgoing wavelengths according to whether they are being used or not by ongoing connections and, if they are being used, according to the incoming link and wavelength over which these connections arrive. The state of the outgoing wavelengths ϕ_1 and ϕ_0 is therefore represented by the pair $\overline{\Lambda} = (\Lambda_1, \Lambda_0)$, where Λ_1 and Λ_0 denote those wavelengths on the incoming links that are being switched to wavelengths ϕ_1 and ϕ_0 respectively on

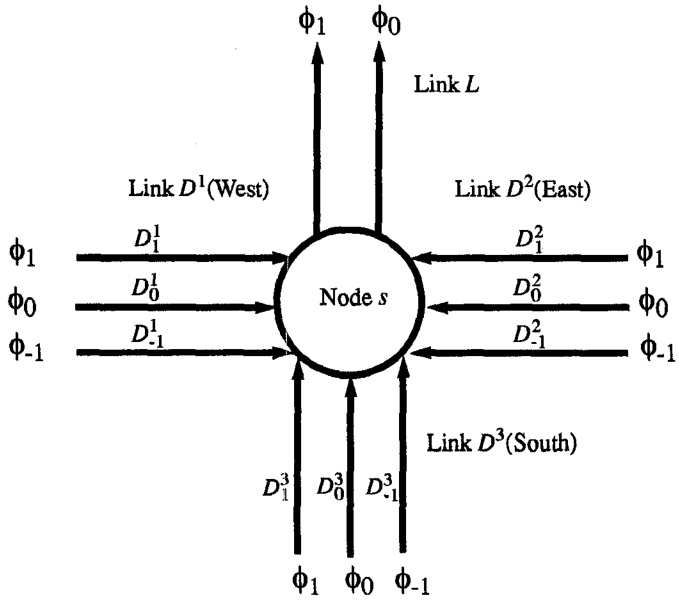

Fig. 2. The relationship of the set of incoming wavelengths to the set of outgoing wavelengths at a node s of the network when 2-adjacent wavelength switching is used. As shown, wavelengths ϕ_{-1} and ϕ_1 on an incoming link may be switched only to wavelength ϕ_0 and ϕ_1 respectively on the outgoing link L, whereas wavelength ϕ_0 on an incoming link can be switched either to wavelength ϕ_0 or to wavelength ϕ_1 on the outgoing link. The labels D_j^i identify the wavelength j and the link D^i on which the setup packet of a session arrives.

link L. A wavelength on link L may also be idle or be used by a new session originating at node s. We will denote by G the state in which a wavelength is being used by a newly generated session at a node, and by I the state in which a wavelength is idle. Based on this, the possible states of wavelengths ϕ_0 and ϕ_1 of link L are

$$\Lambda_0 \in \{D_{-1}^1, D_0^1, D_{-1}^2, D_0^2, D_{-1}^3, D_0^3, I, G\} \stackrel{\text{def}}{=} \mathcal{S}_0, \qquad (1a)$$

and

$$\Lambda_1 \in \{D_0^1, D_1^1, D_0^2, D_1^2, D_0^3, D_1^3, I, G\} \stackrel{\text{def}}{=} \mathcal{S}_1. \qquad (1b)$$

We denote by Ω the set of states of a feasible wavelength set, where

$$\Omega = \{(\Lambda_1, \Lambda_0) \mid \Lambda_1 \in \mathcal{S}_1, \Lambda_0 \in \mathcal{S}_0, \Lambda_0 \neq \Lambda_1, \text{ or } \Lambda_0 = \Lambda_1 = G, \text{ or } \Lambda_0 = \Lambda_1 = I\},$$

and we denote by $\pi(\overline{\Lambda})$ the steady-state probability that a feasible wavelength set on link L is in state $\overline{\Lambda}$. The states (D_0^1, D_0^1), (D_0^2, D_0^2), and (D_0^3, D_0^3) are infeasible, since we consider unicast communication, where an incoming session is switched to a single outgoing wavelength. The states (I, I) and (G, G) however are feasible, and correspond to both outgoing wavelengths being idle or to both being used by originating sessions, respectively. We use the convention that the probability $\pi(\overline{\Lambda})$ of an infeasible state $\overline{\Lambda}$ is zero. The number of states in Ω for a d-dimensional mesh can be found to be $16d^2 - 2d + 1$ when k = 2-adjacent wavelength switching is used, for a total of 61 states for the torus

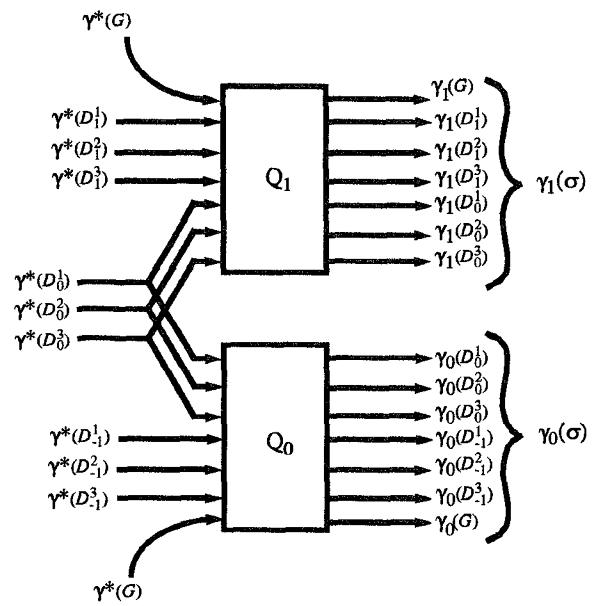

Fig. 3. The auxiliary system $\widehat{\mathcal{Q}}$, which consists of two subsystems $\mathcal{Q}_1$ and $\mathcal{Q}_0$, each consisting of one server and no waiting space. Customers of type σ arrive at the system with rate $\gamma^*(\sigma)$, $\sigma \in \mathcal{S}_l$, $l = 0, 1$, as shown in the figure, and are served according to the rules explained in the text. The rates $\gamma^*(\sigma)$ are chosen so that rate at which customers of type σ depart from subsystem l is $\gamma_l(\sigma)$, $l = 0, 1$.

network ($d = 2$). The case $k = 3$, where an incoming wavelength can be switched to two wavelengths in addition to itself is analyzed in [25]; in that case the number of feasible states of a d-dimensional mesh is $(6d-1)^3-(30d-7)(2d-1)$, for a total of 1172 states for a torus network.

B. The Auxiliary System

We focus on setup packets emitted on wavelength ϕ_0 or ϕ_1 of link L, and we define the *type* σ of a setup packet (and of the corresponding session) according to whether it belongs to a session originating at the node or according to the incoming link and wavelength D_j^i, $i = 1, 2, 3$, $j = 1, 0, -1$, upon which it arrives. Therefore, the set of possible types σ is $\mathcal{S}_0 \cup \mathcal{S}_1 - \{I\}$. We also let $\gamma_l(\sigma)$ denote the rate at which setup packets of type σ are emitted on an outgoing wavelength l, $l = \phi_0, \phi_1$, at a node of the torus. To calculate $\pi(\overline{\Lambda}), \overline{\Lambda} \in \Omega$, we approximate $\pi(\overline{\Lambda})$ as the stationary distribution of an auxiliary system $\widehat{\mathcal{Q}}$ defined as follows (see Fig. 3).

The system $\widehat{\mathcal{Q}}$ consists of two subsystems $\mathcal{Q}_1$ and $\mathcal{Q}_0$, each of which has a single server and no waiting space. Customers of type σ arrive at the system $\widehat{\mathcal{Q}}$ according to a Poisson process with rate $\gamma^*(\sigma)$. Customers arriving at rate $\gamma^*(\sigma)$, $\sigma = D_1^i, i = 1, 2, 3$ ask for the server of $\mathcal{Q}_1$, while those arriving at rates $\gamma^*(\sigma)$, $\sigma = D_{-1}^i, i = 1, 2, 3$ ask for the server of $\mathcal{Q}_0$, and these customers are lost if the corresponding server is busy. Customers of type $\sigma = D_0^i, i = 1, 2, 3$ arrive at rates $\gamma^*(\sigma)$, and they ask for the server of $\mathcal{Q}_0$ or $\mathcal{Q}_1$ with equal probability. If the server

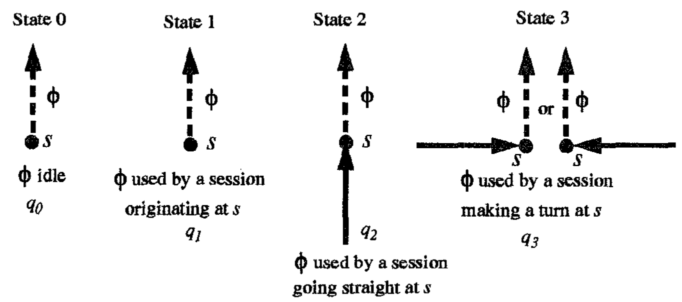

Fig. 4. The four possible states of a wavelength on network link L are illustrated. State 0 corresponds to the case where a wavelength is idle. State 1 corresponds to the case where a wavelength on L is used by a session originating at node s. State 2 corresponds to the case where a wavelength on L is used by a session that is going straight at s. State 3 corresponds to the case where a wavelength on L is used by a session that is making a turn at node s.

they ask for is busy, they ask for the other server, and are dropped only if both servers are busy. Finally, we require that the rate at which customers of type σ are accepted in the auxiliary system $\widehat{\mathcal{Q}}$ be the same as the rate $\gamma_l(\sigma)$ at which setup packets are emitted from an incoming (link, wavelength) pair σ to the outgoing wavelength ϕ_l in the actual system. For this the condition given in Eq. (2) must hold.

C. Calculation of Arrival Rates for the Auxiliary System

In this section, we obtain the arrival rates $\gamma_l(\sigma)$ at which setup packets that arrive over the incoming (link, wavelength) pair σ are emitted over outgoing wavelength ϕ_l. To calculate the rates $\gamma_l(\sigma)$, we observe that each wavelength of a network link can be in one of the four states illustrated in Fig. 4. State 0 corresponds to the case where the wavelength is idle, that is, $\sigma = I$. State 1 corresponds to the case where a wavelength on the outgoing link L at a node s is used by a session originating at node s (a *class 1* session), that is, a session of type $\sigma = G$. State 2 corresponds to the case where a wavelength on L is used by a session that is going straight at node s (a *class 2* session), that is, a session of type $\sigma = D_j^3, j = 0, 1, -1$. Finally, state 3 corresponds to the case where a wavelength on L is used by a session that is making a turn at node s (a *class 3* session), that is, a session of type $\sigma = D_j^i$, $i = 1, 2$ and $j = 0, 1, -1$. Setup packets that place a link in state 1, 2, or 3, will be referred to as being of class 1, 2, or 3, respectively.

In our routing scheme, destinations are distributed uniformly over all nodes (excluding the source node) and no sessions are dropped (recall that blocked sessions are randomly reinserted into the input stream). Thus, wavelength utilization is uniform across all wavelengths of the network, and the probabilities q_0, q_1, q_2, and q_3 that a wavelength is in state 0, 1, 2, or 3 can be obtained simply by counting all possible ways in which a wavelength can be in each of these states and normalizing appropriately. The calculation of the probabilities q_0, q_1, q_2, and q_3 is given in the Appendix. We denote by ρ the total rate at which setup

$$\gamma^*(\sigma) = \begin{cases} \dfrac{2\gamma_l(\sigma)}{\sum_{i \in \mathcal{S}_1, i \neq \sigma} \pi(i, I) + \sum_{j \in \mathcal{S}_0, j \neq \sigma} \pi(I, j) - \pi(I, I)}, & \text{for } \sigma \in \{D_0^1, D_0^2, D_0^3\}; \\ \dfrac{\gamma_0(\sigma)}{\sum_{i \in \mathcal{S}_1} \pi(i, I)}, & \text{for } \sigma \in \{D_{-1}^1, D_{-1}^2, D_{-1}^3\}; \\ \dfrac{\gamma_1(\sigma)}{\sum_{j \in \mathcal{S}_0} \pi(I, j)}, & \text{for } \sigma \in \{D_1^1, D_1^2, D_1^3\}; \\ \dfrac{\gamma_0(\sigma)}{\sum_{i \in \mathcal{S}_1} \pi(i, I)} = \dfrac{\gamma_1(\sigma)}{\sum_{j \in \mathcal{S}_0} \pi(I, j)}, & \text{for } \sigma = G. \end{cases} \quad (2)$$

packets that are finally successful are emitted on an outgoing wavelength of a link. We also denote by ρ_i, $i = 1, 2, 3$ the total rate at which class i setup packets that are finally successful are emitted on a given outgoing wavelength of a link. Clearly,

$$\rho = \rho_1 + \rho_2 + \rho_3. \quad (3)$$

By applying Little's theorem to the system that consists of class i sessions at a particular wavelength, we obtain

$$\rho_i = \frac{q_i}{\overline{X}}, \qquad \text{for } i = 1, 2, 3, \quad (4)$$

where $\overline{X}$ is the mean session holding time.

Applying Little's Theorem to the entire network, and equating the average number of wavelengths used to the product of the average number of active circuits in the system and the mean internodal distance $\overline{h}$, we obtain

$$\rho = \frac{\nu \overline{X}\, \overline{h}}{4W}. \quad (5)$$

The mean internodal distance $\overline{h}$ of the torus can be calculated to be

$$\overline{h} = \begin{cases} p/2, & \text{if } p \text{ is odd;} \\ (p/2)\frac{p^2}{p^2-1}, & \text{if } p \text{ is even.} \end{cases} \quad (6)$$

Referring once again to the actual system depicted in Fig. 2, we see that the rates $\gamma_l(\sigma)$, at which setup packets of type σ are emitted on wavelength ϕ_l, $l = 0$ or 1, of an outgoing link L, can be related in the following way to the rates $\rho_i, i = 1, 2, 3$ at which class i setup packets are emitted on outgoing wavelength l (we write the equations for outgoing wavelength ϕ_0; the case of wavelength ϕ_1 is similar):

$$\rho_3 = \gamma_1(D_0^1) + \gamma_1(D_1^1) + \gamma_1(D_0^2) + \gamma_1(D_1^2),$$

$$\rho_2 = \gamma_1(D_0^3) + \gamma_1(D_1^3),$$

and

$$\rho_1 = \gamma_1(G).$$

Using the symmetry of the rates on the wavelengths of incoming links, we obtain

$$\gamma_l(\sigma) = \begin{cases} \frac{\rho_3}{4}, & \text{for } \sigma \in \{D_1^1, D_0^1, D_{-1}^1, D_1^2, D_0^2, D_{-1}^2\}; \\ \frac{\rho_2}{2}, & \text{for } \sigma \in \{D_1^3, D_0^3, D_{-1}^3\}; \\ \rho_1, & \text{for } \sigma = G. \end{cases} \quad (7)$$

Equation (7) when substituted into Eqs. (2) and (3) gives us the arrival rates for the auxiliary system of Fig. 3. The steady-state probabilities $\pi(\overline{\Lambda})$, $\overline{\Lambda} \in \Omega$, of the auxiliary system (which can be obtained numerically without approximations) are used to approximate the probability that the wavelengths ϕ_0 and ϕ_1 (or, more generally, ϕ_j and ϕ_{j+1}) of the outgoing link L are in state $\overline{\Lambda}$. These probabilities will be used in the next subsection, to obtain the probability that a session successfully establishes a circuit to its destination.

D. Probability of Successfully Establishing a Circuit

The probability that a new session arriving at a random time successfully establishes a circuit from its source node to its destination node depends on its routing tag (t_2, t_1), defined in Section II. We denote by $P_{succ}(t_2, t_1)$ the probability that a new session will succeed in establishing a connection on a particular trial, given that it has a routing tag (t_2, t_1). Using the approach in [26], we will first find an approximate expression for $P_{succ}(t_2, t_1)$, and from it an expression for P_{succ} the probability that a session trying to establish a circuit at a random time is successful.

The path followed by a session with routing tag (t_2, t_1) will make $I(t_2, t_1) - 1$ turns along the way, and will go straight for a total of $|t_2| + |t_1| - I(t_2, t_1)$ steps, where $I(t_2, t_1) \in \{0, 1, 2\}$ is the number of non-zero entries in (t_2, t_1). Thus, the probability α_0 that a wavelength on the first link of the path is available is the probability that the chosen wavelength on the outgoing link is idle, and is given simply by

$$\alpha_0 = \sum_{i \in \mathcal{S}_1} \pi(i, I). \quad (8)$$

At each hop at which a session does not make a turn, the probability that a wavelength ϕ_j or ϕ_{j+1} on the next link L is available given that a wavelength ϕ_j on the previous link $L-1$ used by the session was available, is

$$\begin{aligned} &\Pr\ (\phi_j \text{ or } \phi_{j+1} \text{ on L available} \mid \phi_j \text{ on L} - 1 \text{ available}) \\ &= \frac{\sum_{i \in \mathcal{S}_1, i \neq D_0^3} \pi(i, I) + \sum_{j \in \mathcal{S}_0, j \neq D_0^3} \pi(I, j) - \pi(I, I)}{1 - \sum_{i \in \mathcal{S}_1} \pi(i, D_0^3) - \sum_{j \in \mathcal{S}_0} \pi(D_0^3, j)} \stackrel{\text{def}}{=} \alpha. \end{aligned} \quad (9)$$

Similarly, at each hop at which a session makes a turn, the probability that a wavelength ϕ_j or ϕ_{j+1} on the next link L is available given that a wavelength ϕ_j on the previous link $L-1$ used by the session was available, is

$$\Pr\left(\phi_j \text{ or } \phi_{j+1} \text{ on L available} \mid \phi_j \text{ on L}-1 \text{ available}\right) = \frac{\sum_{\substack{i\in\mathcal{S}_1\\ i\neq D_0^1}} \pi(i,I) + \sum_{\substack{j\in\mathcal{S}_0\\ j\neq D_0^1}} \pi(I,j) - \pi(I,I)}{1-\sum_{i\in\mathcal{S}_1}\pi(i,D_0^1)-\sum_{j\in\mathcal{S}_0}\pi(D_0^1,j)} \stackrel{\text{def}}{=} \beta. \quad (10)$$

In writing Eqs. (9) and (10) above, we do *not* assume that the probabilities of acquiring wavelengths on successive links of a session's path are independent. Instead, we account partially for the dependence between the acquisition of successive wavelengths on a session's path by using the approximation that the probability of acquiring a wavelength on link L depends on the availability of the corresponding wavelength on link $L-1$ (in reality this probability depends, even though very weakly, on the availability of corresponding wavelengths on all links $1, 2, \ldots, L-1$ preceding link L). As the simulation results presented in the next section demonstrate, our approximation is a very good one, however.

The (conditional) probability of successfully establishing a connection is then given by

$$P_{succ}(t_2, t_1) = \alpha_0 \cdot \alpha^{|t_2|+|t_1|-I(t_2,t_1)} \cdot \beta^{I(t_2,t_1)-1},$$

which, for uniformly distributed destinations, gives

$$P_{succ} = \frac{1}{p^2-1}\sum_{t_2,t_1} P_{succ}(t_2,t_1).$$

After some algebra, the above expression yields [26]

$$P_{succ} = \frac{\alpha_0}{\beta(p^2-1)}\left[\left(1+2\beta\left(\frac{1-\alpha^D}{1-\alpha}\right)\right)^2-1\right] \quad (11)$$

for p odd, and

$$P_{succ} = \frac{\alpha_0}{\beta(p^2-1)} \times \left[\left(1+\beta\left(\frac{1-\alpha^D}{1-\alpha}\right)+\beta\left(\frac{1-\alpha^{D-1}}{1-\alpha}\right)\right)^2-1\right] \quad (12)$$

for p even, where $D = \lfloor p/2 \rfloor$, and α_0, α and β are given by Eqs. (8), (9), and (10), respectively. The probability that a session is blocked is therefore given by

$$P_{blk} = 1 - P_{succ}.$$

III. Results and Discussion

In this section, we present simulation and analytical results for the torus network for three different cases: the case of no wavelength translation (or 1-adjacent wavelength switching); the case of limited wavelength translation using k-adjacent wavelength switching, where $k = 2, 3$; and the case of full wavelength translation (or W-adjacent wavelength switching) in a WDM network with W wavelengths per link (fiber). We note that full-wavelength translation provides the best achievable performance (in terms of the realizable probability of success for a given arrival rate per wavelength, or in terms of the realizable throughput per wavelength for a given probability of success) for a given number of wavelengths W per fiber.

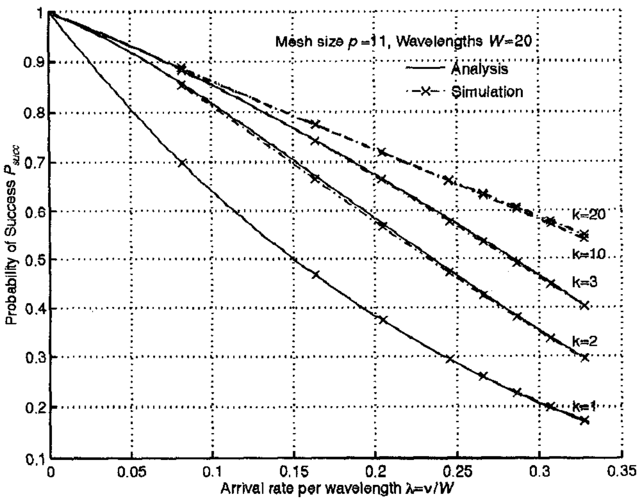

Fig. 5. The probability of successfully establishing a connection P_{succ} versus the arrival rate per wavelength λ, for a mesh of size $p = 11$, for $W = 20$ wavelengths per link. The plots show the analytically predicted values and simulation values of P_{succ} for the case of 1-adjacent wavelength switching ($k = 1$; no wavelength translation), for the case of 2-adjacent wavelength switching ($k = 2$; limited wavelength translation), and for the case of W-adjacent wavelength switching ($k = W$; full-wavelength translation). All calculations and simulations have been done with session holding times exponentially distributed with mean $\overline{X} = 1.0$.

When no wavelength translation is used, the different wavelengths on a link do not interact with one another, and an all-optical network with W wavelengths per fiber is essentially equivalent to W disjoint single-wavelength networks operating in parallel. To obtain the probability of success in this case, it is therefore enough to focus attention on any one of the W independent parallel networks, for which the analysis given in [26] applies.

In Figs. 5 and 6 we present the results comparing our analysis and simulations for the probability of success P_{succ} plotted versus the arrival rate per node per wavelength $\lambda = \nu/W$ when limited wavelength translation to only one or two additional wavelengths (i.e, $k = 2, 3$) is permitted. The analytical results for $k = 3$-adjacent wavelength switching shown here, were obtained using the analysis presented in [25]. Each point in our simulation was obtained by averaging over 2×10^6 successes. We note that our results are very accurate, with the analysis and simulations agreeing to within 1%. Observe that, for the range of rates shown, limited translation to only one or two adjacent wavelengths provides a considerable fraction of the improvement that full wavelength translation provides over no wavelength translation.

Observe also that the benefits of wavelength translation diminish as the extent of translation k increases, and eventually appear to saturate. We see therefore that our analysis and simulations predict that limited translation of small range, i.e., $k = 2$ or 3, gives most of the benefits obtained by full wavelength translation, where $k = W$. See for instance

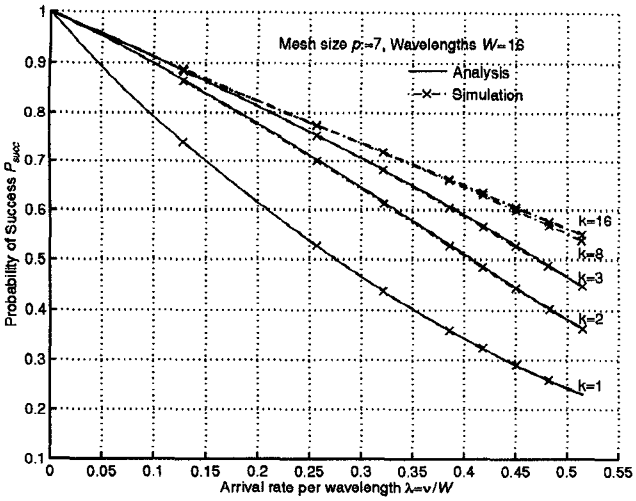

Fig. 6. The probability of successfully establishing a connection P_{succ} versus the arrival rate per wavelength λ, for a mesh of size $p = 7$, for $W = 16$ wavelengths per link. The simulation parameters are the same as those given in Fig. 5.

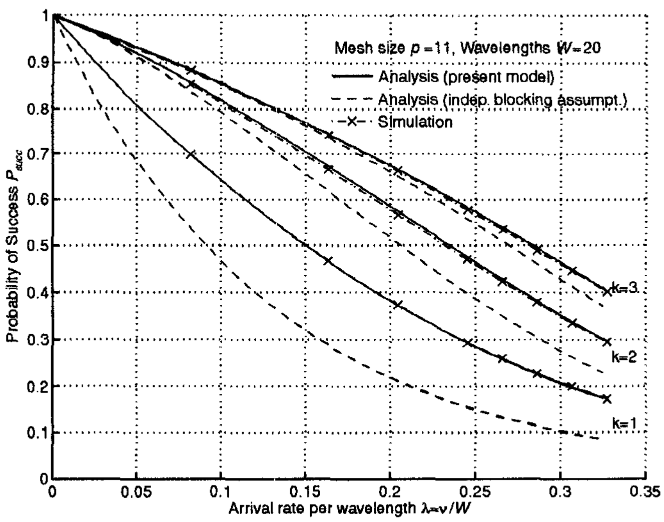

Fig. 7. Comparing the probability of success P_{succ} for the torus network, obtained by using the link independence blocking assumption and the present analysis. For small degree networks, like the mesh, The independence assumption is seen to break down at larger loads, where as the present analysis accurately captures network performance over the entire range of loads.

Figs. 5 and 6, which also illustrate the network performance for $k = W = 20$ and for $k = W = 16$ wavelengths, respectively. As is evident from the plots, increasing the extent of translation k beyond some value leads to diminishing returns. In fact, we conjecture that increasing the extent of translation k beyond the network diameter, will result in only a negligible increase in the probability of success.

By contrast, using the model that they had considered, Yates et al [28] had predicted based on their simulations that limited wavelength translation of degree $\pm 50\%$ (which corresponds approximately to $k = (W-1)/2$ in our notation, since Yates et al define the degree of translation by the number of translations allowed on *either side* of the input wavelength and do not permit wraprounds) would be needed to give performance nearly equal to that of full wavelength translation. Yates et al, however, obtained improved results by using a model where a new session attempts to establish a connection by trying different starting wavelengths, and, in fact, by examining all alternate routes emanating from each of those wavelengths. Such improvements are based essentially on trying multiple options simultaneously, or, alternatively, on using backtracking while establishing the connection, which would not be practical in many cases in view of roundtrip delays and the blocking of other sessions that will occur during the setup phase, if it is done through multiple setup packets. The improvements obtained by trying different options increase with increasing W and k. They increase with k because at each node a session examines all k options (which would be impractical when the roundtrip delay is comparable to session holding time, but could be practical otherwise), and they increase with W because at the first hop a session examines all W wavelengths on its outgoing link. Indeed if the link utilization information available at a source is accurate and connection setup takes zero time (that is, the roundtrip delay ≈ 0), then the results of Yates' et al correspond to finding a path when running the routing algorithm, and it would be possible in this case to gain even further by trying not only alternative wavelengths, but also alternative topological paths from the source to the destination.

Our model, on the other hand, considers the more practical (in many cases) situation, where, at its first hop, a new session selects a wavelength upon which to attempt and at each subsequent hop looks only at the availability of the wavelengths at that hop and not of those beyond, and therefore does not try to "see the future," as it were. Our analysis illustrates that a favorable trade-off may result between the extent of translation k and the fraction of the performance of full wavelength translation that is achievable by using limited wavelength translation. Furthermore, as is evident from the results presented in Figs. 5 and 6, for a large range of network loads, limited translation of relatively small degree suffices to give performance comparable to that obtained by full wavelength translation.

In Fig. 7 we compare the probability of success obtained via our analytical and simulation results with that obtained by using the link independence blocking assumption, for the torus network. As is evident from the plots, the present analysis is very accurate for a large range of networks loads, including heavy loads; the independence blocking assumption, however, becomes inaccurate at heavy loads. The significant difference for the mesh between the present analysis and an analysis using the independence blocking assumption can be attributed to the small node degree and large diameter (and mean internodal distance) of the mesh. A small degree leads to less mixing of traffic and therefore a non-negligible dependence between the probability of acquisition of successive wavelengths on the path followed by the setup packet of a session. A large diameter tends to amplify the inaccuracy in the probability of acquisition of each link, since some paths are long.

IV. Conclusions

We examined the case of limited wavelength translation in wavelength routed, all-optical, WDM mesh networks, and demonstrated that limited wavelength translation of fairly small degree is sufficient to obtain benefits comparable to those obtained by full wavelength translation. An important conclusion of our analysis was that the *efficiency* with which capacity is used depends on the extent of translation k and not on the total number of wavelengths W per fiber, as long as $W \geq k$ (equivalently, the throughput of the network for a given blocking probability increases with k, and increases only linearly, and not superlinearly, with the number of wavelengths W). Increasing W with k fixed does not increase the probability of success P_{succ} for the same value $\lambda = \nu/W$ of admissible throughput per node per wavelength. This seems at first view to contrast with the observation made in [24], where the authors had found that for sparse wavelength conversion, converters help more (in improving efficiency) the more wavelengths there are per fiber. The model used in [24], however, essentially relies on back tracking or trying multiple options simultaneously, which may not be a feasible option in WANs with large propagation delays, as discussed in Section III. Our results also indicate that the network performance improves as the extent of translation k increases, but that the rate of improvement typically decreases with increasing k, and becomes negligible when k is equal to the diameter of the network.

Our analysis also answers partially the question raised in [26], namely, would the throughput of a circuit-switched mesh network with k ($k > 1$), circuits per link be expected to improve by more than just a factor of k? Since the success probability in our case depends on the extent of translation k and is independent of the number of wavelengths W per link ($W \geq k$), we can also view the P_{succ} curves in Figs. 5 and 6 for $k = 2, 3$ as representing the probability of successfully establishing a connection in a circuit switched mesh network with a capacity of k circuits per link. We observe that with $k = 2, 3$ circuits per link, the improvement in throughput over a network with a capacity of only 1 circuit per link is considerably greater than by just a factor of k. (If the improvement obtained was only a factor of k, the curves for $k = 2$ and $k = 3$ would overlap the curves for $k = 1$, which is not the case.) However, increasing k further, say from 2 to 3, gives diminishing returns. This observation agrees with the result obtained by Koch [15], who proved that the maximum total throughput of a $d2^d$-node butterfly with a capacity of k circuits per link is $\Theta(2^d/d^{1/k})$, and, therefore, that the increase in efficiency obtainable by using links with larger capacity diminishes as the capacity k per link increases.

Finally, our analysis was based on the use of fixed shortest path routing to establish a connection. The effect of alternate or dynamic routing when combined with limited wavelength translation is a possible area for future work. Furthermore, as observed in [24], the success probability is only one measure of network performance. A study that looks at other measures of network performance, such as the input queueing and connection delays, to evaluate the benefits of limited wavelength translation would also be useful.

V. Appendix: Derivation of wavelength state probabilities

A. *Derivation of the probabilities $q_i, i = 1, 2, 3$ for the torus*

In this appendix, we evaluate the probabilities $q_i, i = 1, 2, 3$ for the torus or 2-dimensional wrap-around mesh network. We treat separately the cases p odd and p even.

(a) **Case p odd :**
The probability q_3 that a wavelength on an outgoing link L is used by a turning session is given by

$$q_3 = K \sum_{\substack{t_2=-\lfloor p/2 \rfloor \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} \sum_{\substack{t_1=-\lfloor p/2 \rfloor \\ t_1 \neq 0}}^{\lfloor p/2 \rfloor} 1 = K(p-1)^2. \qquad (A.1)$$

Similarly, the probability q_2 that a wavelength on an outgoing link L is used by a straight-through session is given by

$$q_2 = K\Bigg[\sum_{\substack{t_2=-\lfloor p/2 \rfloor \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} \sum_{\substack{t_1=-\lfloor p/2 \rfloor \\ t_1 \neq 0}}^{\lfloor p/2 \rfloor} (|t_2| + |t_1| - 2) + \binom{2}{1} \sum_{\substack{t_2=-\lfloor p/2 \rfloor \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} (|t_2| - 1) \Bigg] \qquad (A.2)$$

After some algebra, Eq. (A.2) reduces to

$$q_2 = K\frac{p(p-1)(p-3)}{4}. \qquad (A.3)$$

The remaining probability q_1 that a wavelength on link L is used by a session originating at that node is given by

$$q_1 = K(p^2 - 1). \qquad (A.4)$$

(b) **Case p even:**
As before, the probability q_3 that a wavelength on an outgoing link L is used by a turning session is given by

$$q_3 = K \sum_{\substack{t_2=-(\lfloor p/2 \rfloor - 1) \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} \sum_{\substack{t_1=-(\lfloor p/2 \rfloor - 1) \\ t_1 \neq 0}}^{\lfloor p/2 \rfloor} 1 = K(p-1)^2. \qquad (A.5)$$

The probability q_2 that a wavelength on an outgoing link L is used by a straight-through session is given by

$$q_2 = K\Bigg[\sum_{\substack{t_2=-(\lfloor p/2 \rfloor - 1) \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} \sum_{\substack{t_1=-(\lfloor p/2 \rfloor - 1) \\ t_1 \neq 0}}^{\lfloor p/2 \rfloor} (|t_2| + |t_1| - 2) + \binom{2}{1} \sum_{\substack{t_2=-(\lfloor p/2 \rfloor - 1) \\ t_2 \neq 0}}^{\lfloor p/2 \rfloor} (|t_2| - 1) \Bigg] = Kp\frac{(p-2)^2}{2}. (A.6)$$

As before, the probability q_1 is given by Eq. (A.4). Finally, a simple application of Little's Theorem gives

$$(1-q_0)\cdot(4p^2)W = p^2\nu\overline{X}\,\overline{h},$$

or

$$1-q_0 = \frac{\nu\overline{X}\,\overline{h}}{4W}.$$

These relationships combined with $\sum_{i=0}^{3} q_i = 1$ give

$$K = \frac{1-q_0}{p\cdot\lceil\frac{p^2-1}{2}\rceil}, \qquad (A.7)$$

which when combined with Eqs. (A.1) – (A.6) gives

$$q_1 = \frac{\nu\overline{X}\,\overline{h}}{4W}\cdot\frac{p^2-1}{p\cdot\lceil\frac{p^2-1}{2}\rceil}, \qquad (A.8a)$$

$$q_2 = \frac{\nu\overline{X}\,\overline{h}}{4W}\cdot\frac{\lceil\frac{p-2}{2}\rceil\lfloor\frac{p-2}{2}\rfloor}{p\cdot\lceil\frac{p^2-1}{2}\rceil}, \qquad (A.8b)$$

and

$$q_3 = \frac{\nu\overline{X}\,\overline{h}}{4W}\cdot\frac{(p-1)^2}{p\cdot\lceil\frac{p^2-1}{2}\rceil}, \qquad (A.8c)$$

where $\lfloor x\rfloor$ is the largest integer less than or equal to x, and $\lceil x\rceil$ is the smallest integer greater than or equal to x. Equations (A.8a)-(A.8c) hold for both p odd and p even.

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