Power-induced time division on asynchronous channels

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Time division multiple access offers certain well-known advantages over methods such as spread spectrum code division. Foremost is the interference immunity provided by dedicated time slots. Partly offsetting this is TDMA's need for network-wide synchronization. Viewing arbitrary time intervals as potential TDMA time slots, we ask whether it is possible to obtain some of the benefit of time division without incurring the synchronization cost. In particular, we address the question of whether a TDMA-like state can be induced on asynchronous channels in such a way as to reduce interference and energy consumption. Through analysis and simulation we find conditions under which it is beneficial to use time division, and then show how autonomous power management may be used as a mechanism to induce a form of time division. In this context a backlog-sensitive power management system is presented.

1. Introduction

A considerable amount of recent research attention and standards activity has been focused on the question of whether code division or time division is better for wireless networks [2,4,6]. In fact, the lack of a single standard has been identified as "the biggest hurdle for the wireless revolution" [12]. As things now stand, the GSM/IS-54/JDC Time Division (TDMA) and Qualcomm/IS-95 Code Division (CDMA) multiple access schemes are competing, and incompatible at the MAC and physical layers.[1] Two (of several) primary factors in the competition are interference management and energy consumption, and by elementary examples included below one can see that under some conditions code division yields preferred interference/energy characteristics, while for others time division is better. While this may not be particularly surprising, it is interesting for the question it raises: Can a single network effectively switch itself between modes in some way so as to take advantage of the preferred multiple access method? Better still, can this switching be achieved without the intervention (and added complexity and cost) of a central controller?

It turns out that conventional transmitter power control [1,5,7–10,13–15,21–24] can be adapted to the task. In section 2, we briefly explain the conventional, constant-SIR method of power control (CSPC), and show by a simple example how an alternative method can lead to substantial reduction in overall energy consumption. Then, in section 3, we examine a fundamental stability criterion (achievability) in the context of CSPC and our alternative, which we associate with time division multiple access (TDMA), and discuss how to detect which mode – time-divided or not – is preferred when target throughputs are achievable either way. Next, in sections 4 and 5, we review interference-based power *management*, introduce a form of backlog-sensitive power management, and show how the two can be integrated. The aim is to show that it is possible to achieve some of the benefits of TDMA in a code division multiple access (CDMA) or other non-TDMA system, through the use of power management. In section 6, we discuss simulation results which show the potential strengths and shortcomings of the proposed system. Some concluding remarks are included in section 7.

[1] GSM is Global System for Mobile Communications. IS-54 and IS-95 (Interim Standards 54 and 95) are North American industry standards. JDC stands for Japanese Digital Cellular, and MAC for medium access control.

2. Power-controlled wireless networks

Consider a network with N mobile transmitters, each communicating with another node or nodes over wireless channels, with the channels separated by codes, space, and/or frequency. For i and j in $\{1,\dots,N\}$ let $g_{i,j}$ represent the interfering-link gain from transmitter j to i's receiver when $j \neq i$, and the intended-link gain (including path loss and signal processing gain) between i and i's receiver when $j = i$, so a signal transmitted by j with power p_j is received by i's receiver with power $g_{i,j}p_j$. Define the $N\times N$ gain matrix $G = [g_{i,j}]$. In addition, denote by ξ_i the noise power at i's receiver, and by $\mathcal{P}_i$ the set of admissible power functions for transmitter i.

Given such a network and some power control objective, we call the objective *achievable* iff it is satisfied for a set of power functions $\{p_i \in \mathcal{P}_i\colon\ i = 1,\dots,N\}$. For instance, the objective may be to attain a set of N specified signal to interference ratios (SIRs) γ_i, one for each transmitter/receiver pair. If also each $\mathcal{P}_i$ is the set of nonnegative real-valued constant functions, we have a constant-SIR power-controlled (CSPC) system. Then in vector/matrix notation (abbreviating subscripts "i,j" to "ij") we desire a solution to the equation

$$p = \widehat{Z}p + \widehat{\xi}, \qquad (1)$$

where

$$\widehat{Z} = \widehat{G} - \text{diag}(\gamma), \quad \widehat{G}_{ij} = \frac{\gamma_i g_{ij}}{g_{ii}}, \quad \widehat{\xi}_i = \frac{\gamma_i \xi_i}{g_{ii}}.$$

The objective SIR vector $(\gamma_1, \ldots, \gamma_N)$ is achievable iff the determinant $|I - \widehat{Z}|$ is positive, in which case the solution is given by

$$p = \left(I - \widehat{Z}\right)^{-1}\widehat{\xi}. \quad (2)$$

For instance, the solution exists for $N = 2$ iff

$$\gamma_1\gamma_2 \frac{g_{12}g_{21}}{g_{11}g_{22}} < 1. \quad (3)$$

When we can confirm that a network objective is achievable, our attention turns to whether there exists an algorithm to realize the desired state, over time, starting from a given initial condition. In particular, we are interested in algorithms for selecting transmitter powers p_i^{n+1} at times $n = 0, 1, \ldots$, and starting from p_i^0. Let us say that a network, together with an algorithm (or collection of algorithms), is *power-stable* iff the algorithm generates a sequence of admissible power vectors $\{p^n\colon\ p_i^n \in \mathcal{P}_i^n,\ n = 1, 2, \ldots\}$ such that for all i,

$$p_i^n \to p_i \in \mathcal{P}_i, \quad n \to \infty,$$

and such that the objective is satisfied for the limiting powers $\{p_i\}$. For example, let us write γ_i^n for the SIR at time n on channel i. Then Foschini and Miljanic [7] have shown that whenever a CSPC system (as described above) with constant noise power vector ξ has an achievable vector γ, and the spectral radius of $\widehat{Z}$ is less than one, the network/algorithm

$$p_i^{n+1} = \frac{\gamma_i}{\gamma_i^n} p_i^n \quad (4)$$

is power-stable; that is, the power vector converges to a value which yields the target SIR vector γ.

For the remainder of this section let us focus on the case $N = 2$. If the ith channel's receiver noise is ξ_i^n at time n then the SIR on channel i is

$$\gamma_i^n = \frac{g_{ii}p_i^n}{g_{ij}p_j^n + \xi_i^n}.$$

Given the error rate as a function of SIR, we can calculate the successful data transmission rate as a function of these parameters. Typical bit error rates (BERs) for a number of common modulation schemes and environments are well known, such as binary differential phase shift keying (DPSK) in a non-fading environment (BER $a\exp\{-b\gamma\}$ for SIR γ and, e.g., $a = b = 1$) or binary non-coherent frequency shift keying in the presence of fading (BER $1/(\gamma + 2)$) [16]. The function

$$e = \frac{1}{1 + \gamma} \quad (5)$$

represents a reasonable upper bound on the bit error rate (BER) relative to BERs obtained with many conventional modulation schemes. Assume for now that equation (5) holds. (Power control operation and performance will be sensitive to the form of e, so we consider other forms of the BER function below.) In order to transmit at respective rates or throughputs r_1 and r_2 ($0 \leqslant r_i < 1$) the transmitter powers would have to be sufficient to attain receiver SIRs of $\gamma_1 = r_1/(1 - r_1)$ and $\gamma_2 = r_2/(1 - r_2)$, respectively. Solving for p_1 and p_2 simultaneously in terms of rates and gains we obtain

$$p_1 = \frac{r_1(g_{12}r_2\xi_2 + g_{22}(1 - r_2)\xi_1)}{(g_{11}g_{22} - g_{12}g_{21})r_1r_2 + g_{11}g_{22}(1 - r_1 - r_2)} \quad (6)$$

and

$$p_2 = \frac{r_2(g_{21}r_1\xi_1 + g_{11}(1 - r_1)\xi_2)}{(g_{11}g_{22} - g_{12}g_{21})r_1r_2 + g_{11}g_{22}(1 - r_1 - r_2)}, \quad (7)$$

as long as the rates r_i are achievable, i.e.,

$$\frac{r_1}{1 - r_1}\frac{r_2}{1 - r_2}\frac{g_{12}g_{21}}{g_{11}g_{22}} < 1.$$

Using this constant-SIR transmission scheme, the total energy used to obtain rates r_1 and r_2 over a unit time interval is therefore

$$\mathcal{E} = \frac{r_1\xi_1(g_{21}r_2 + g_{22}(1 - r_2)) + r_2\xi_2(g_{11}(1 - r_1) + g_{12}r_1)}{(g_{11}g_{22} - g_{12}g_{21})r_1r_2 + g_{11}g_{22}(1 - r_1 - r_2)}.$$

The immediate question is: Can the same rates be obtained using less energy?

Certainly the answer is “no” under CSPC; the solution to (1) is unique for non-zero receiver noise powers. However, it is possible, given certain values of the gain matrix G and rates r_i, to save energy by switching to a time-division-like access protocol.

Example 1. Consider the following parameters, which might represent reasonable values in a cellular code division multiple access (CDMA) system: $r_1 = r_2 = 1/3$, $g_{11} = g_{21} = 1$, $g_{22} = g_{12} = 1/16$, $\xi_1 = \xi_2 = 1$. The values obtained using (6) and (7) are

$$p_1 = 1 \quad \text{and} \quad p_2 = 16.$$

If the time period of interest is the unit interval, then the transmitters will use 1 and 16 units of energy, respectively, for a total of $\mathcal{E} = 17$, the total area under the constant-power curves of figure 1.

On the other hand, if the two transmitters were permitted *piecewise* constant power control functions, and used distinct time segments – only transmitter one during $(0, \tau]$, only transmitter two during $(\tau, 1]$ – then it would be possible to lower the total energy used. For instance, if $\tau = 2/5$ then

$$p_1(t) = \begin{cases} 5, & 0 < t \leqslant 2/5, \\ 0, & 2/5 < t \leqslant 1, \end{cases}$$

and

$$p_2(t) = \begin{cases} 0, & 0 < t \leqslant 2/5, \\ 20, & 2/5 < t \leqslant 1, \end{cases}$$

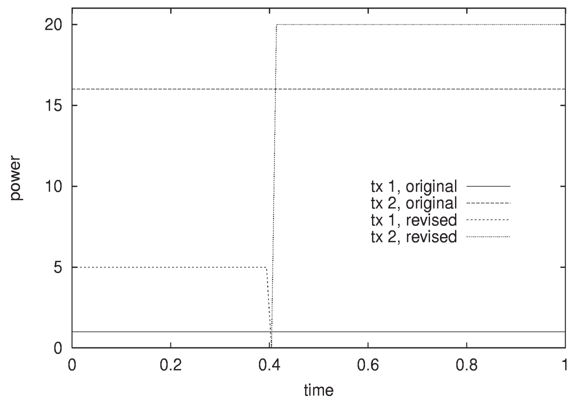

Figure 1. Power levels for two transmitters under the CSPC (original) and time division (revised) systems.

would also yield rates $r_1 = r_2 = 1/3$ but would consume only $\mathcal{E} = 14$ units of energy, versus 17 under CSPC. We use the term "induced TDMA" (iTDMA) for power control algorithms which produce this effect.

In the preceding example there is a nearly twenty percent reduction in energy usage under induced TDMA. Moreover, the lower total transmitter power means lower interference to other users, and hence potential benefits like improved network throughput and stability. What are the drawbacks? First, one of the transmitters is called on to *increase* its energy consumption from 1 to 2 in switching to the more globally efficient time division scheme; this is likely to be unsatisfactory if the user is not eventually compensated. Second, the asynchronous static scheme is replaced by a synchronized system wherein each transmitter must know when to turn on and off, meaning increased system complexity.

While the first drawback is inherent to the protocol, the second can be at least partially avoided. In fact, it may be possible to reap much of the benefit usually associated with TDMA without using central control or synchronization to force users into time slots. The idea is to have sufficient incentive for the users to automatically detect conditions favorable to a TDMA structure and implement them *without* the intervention of a base station or other mediator. One such incentive may be provided by power management, to which we will turn shortly.

Example 1 illustrates the potential benefits of inducing time division. The next question is: When is it the right thing to do?

3. When is time division preferred?

In example 1 we used transmission rates rather than SIRs as our performance objective. While the two are usually related in a one-to-one manner (rates are usually increasing functions of SIR), focusing on the rate and letting the SIR vary over the interval of interest allowed us to consider non-static solutions. This, in turn, makes induced time division power control systems possible. Hence the question of interest becomes: When, in general, is it better to use iTDMA than CSPC? In terms of access protocols, this is akin to asking: When is TDMA better than pure CDMA?[2]

For an iTDMA system, one easily finds that the rates $\{r_i\}$ are achievable iff

$$\sum_{i=1}^{N} \frac{r_i}{1 - e_i(p_i)} < 1,$$

where e_i is the error rate for channel i during i's slot, i.e., the portion of time that transmitter i is alone using non-zero power. If we assume (as we have so far) that the admissible power functions are unbounded above then e_i can be made arbitrarily close to 0 and the achievability condition becomes the intuitively satisfying expression

$$\sum_{i=1}^{N} r_i < 1.$$

Before we turn to the general CSPC achievability condition, let us look at a special case which will set the stage.

Example 2. Let us again restrict our scope to the case $N = 2$. From the above we know the rates r_1 and r_2 are achievable under iTDMA iff

$$r_1 + r_2 < 1. \tag{8}$$

Now consider the CSPC system. If we let $e_i = e$ as given in (5) then the achievability condition is (3); writing it to facilitate comparison with iTDMA,

$$\frac{r_1 r_2}{1 - (r_1 + r_2) + r_1 r_2} < \frac{g_{11} g_{22}}{g_{12} g_{21}}. \tag{9}$$

Now it is a consequence of physical conditions existing in typical systems that, for all $i = 1, \ldots, N$, either

$$g_{ii} \geqslant g_{ji} \quad \text{for all } j \quad \text{or} \quad g_{ii} \geqslant g_{ij} \quad \text{for all } j, \tag{10}$$

or both. Indeed, in a code-separated system (such as CDMA) one would expect the first condition to hold (loosely, i's signal will be strongest at i's intended receiver, since they use the same pseudo-noise code sequence), while in a frequency- and/or space-separated system one would expect both conditions in (10) to hold. (The second condition could conceivably be violated in a CDMA system due to near-far effect.) It follows from (10) that

$$\frac{g_{11} g_{22}}{g_{12} g_{21}} \geqslant 1. \tag{11}$$

It then follows easily from (11) and conditions (8) and (9) that if the rates (r_1, r_2) are achievable under iTDMA, then they are achievable under the CSPC. The converse is not

[2] More generally, when is a time division system better – at least in terms of power and interference control – than a non-time-division system? Note that real systems almost always include some implicit use of frequency division (FDMA), so CDMA is really CDMA/FDMA, etc. To be precise, we might say we are comparing CDMA/FDMA with CDMA/FDMA/TDMA.

true (consider $r_1 + r_2 \geqslant 1$ with g_{12} and g_{21} sufficiently small).

We can follow reasoning from example 2 to the proposition below. Let $\gamma_i(r)$ be the SIR necessary to achieve rate r on channel i.

Proposition. The rate vector $(r_1, \ldots, r_N)$ is achievable under iTDMA iff

$$\sum_{i=1}^{N} r_i < 1.$$

Under the assumption (10), the same rate vector is achievable under CSPC if $|\Gamma| > 0$, where

$$\Gamma = \begin{bmatrix} 1 & -\gamma_1(r_1) & -\gamma_1(r_1) & \ldots & -\gamma_1(r_1) \\ -\gamma_2(r_2) & 1 & -\gamma_2(r_2) & \ldots & -\gamma_2(r_2) \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -\gamma_N(r_N) & \ldots & -\gamma_N(r_N) & -\gamma_N(r_N) & 1 \end{bmatrix}.$$

Proof. The first statement is immediate. To prove the second statement, observe that $I - \widehat{Z}$ may be obtained from Γ by multiplying all off-diagonal elements Γ_{ij} by g_{ij}/g_{ii}. The determinant of Γ takes the form $1 - S$, where S is a sum of $N! - 1$ terms of the form

$$\gamma_{i_1} \cdots \gamma_{i_n}, \tag{12}$$

$1 < n \leqslant N$. Likewise, the determinant of $I - \widehat{Z}$ takes the form $1 - T$, where T is an $N! - 1$ term sum such that to each term (12) of S there corresponds a term of T given by

$$\gamma_{i_1} \cdots \gamma_{i_n} \frac{g_{i_1 i_{j_1}} \cdots g_{i_n i_{j_n}}}{g_{i_1 i_1} \cdots g_{i_n i_n}}, \tag{13}$$

with $(j_1, \ldots, j_n)$ a permutation of $(i_1, \ldots, i_n)$. Under assumption (10), the fractional factor in (13) is less than unity. It follows that $T \leqslant S$ and thus

$$\left|I - \widehat{Z}\right| = 1 - T \geqslant 1 - S = |\Gamma|.$$

We recall from section 2 that $|I - \widehat{Z}| > 0$ is necessary and sufficient for achievability of the SIR vector γ. Thus $|\Gamma| > 0$ is sufficient for the vector of SIRs $(\gamma_1(r_1), \ldots, \gamma_N(r_N))$ to be achievable. □

In situations where the objective is achievable under both iTDMA and CSPC, it is worth exploring which system will operate at lower total power. The first step is to find out how to optimally divide the interval among the users. If the optimal solution yields a total energy greater than that used under CSPC, there is no reason to continue with the induced time division approach. Let us examine how the optimal time partition is calculated.

Assuming the rates $(r_1, \ldots, r_N)$ are achievable for iTDMA, we wish to find the values t_i, $i = 1, \ldots, N$, which solve

$$\min \sum_{i=1}^{N} p_i t_i \tag{14}$$

subject to

$$t_i\bigl(1 - e_i(p_i)\bigr) \geqslant r_i, \quad i = 1, \ldots, N, \tag{15}$$

and

$$\sum_{i=1}^{N} t_i = 1.$$

Again assuming e_i is known and is strictly decreasing for each i, we can use Lagrange's method to find the t_i; or in simpler cases, if we can take equality in the equations (15) and solve each for p_i in terms of t_i, then we can plug the results into the objective (14) and use ordinary calculus to find the optimal t_i.

Example 3. For $N = 2$ let $t_1 = \tau$ and $t_2 = 1 - \tau$. The constraints become

$$\tau\bigl(1 - e_1(p_1)\bigr) \geqslant r_1 \tag{16}$$

and

$$(1 - \tau)\bigl(1 - e_2(p_2)\bigr) \geqslant r_2. \tag{17}$$

Taking equality in (16) and (17) under (5), solving for p_1 and p_2, and plugging into the objective function gives the expression

$$\tau = \frac{r_1 r_2^2 \tilde{\xi}_2 - r_1^2(1 - r_2)\tilde{\xi}_1 \pm r_1 r_2 (1 - r_1 - r_2)\sqrt{\tilde{\xi}_1 \tilde{\xi}_2}}{r_1^2 \tilde{\xi}_1 r_2^2 \tilde{\xi}_2},$$

where $\tilde{\xi}_i = \xi_i / g_{ii}$. We also obtain

$$\frac{p_1}{p_2} = \sqrt{\frac{g_{22}\xi_1}{g_{11}\xi_2}}.$$

These equations may be used to show that the induced time division scheme of example 1 is optimal, with $\tau = 2/5$.

Once the t_i are known the minimum energy can be calculated. This value can then be compared to the minimum CSPC energy calculated from (2). If the energy-optimal time division scheme uses less energy than CSPC, there is incentive to use iTDMA. In this case it will be of interest to find an efficient way to induce the TDMA effect. To put this another way, the calculations described in this section give us a prescription for determining whether and when we may improve upon asynchronous CSPC operation. We turn now to power management for some ideas on how improvement may be achieved without a central controller.

4. Interference-based power management

Conventional power control schemes such as CSPC call for constant or increased power when increased interference is encountered. By contrast, power management, i.e., power control for battery life maximization [17,19], typically dictates that power be *decreased* if interference exceeds certain thresholds. Given this, one can envision two or more power-managed, mutually interfering transmitters effectively taking turns accessing a channel. Before we elaborate, let us summarize some earlier work on power management.

The power management problem may be described as selecting transmitter power as a function of receiver interference and possibly other information in a way that minimizes the rate of energy consumption, subject to various quality-of-service (QOS) constraints; for a general and more complete description than is given below, the reader is referred to [19]. Let us first consider a discrete-time system such that the receiver interference in successive time instants forms an independent, identically distributed (i.i.d.) positive-integer-valued stochastic sequence independent of the transmitter power. One may then write $P_I(i)$ for the probability that interference is i, and express the power management problem in the form

$$\begin{aligned} &\min_p \sum_{i=1}^{\infty} p(i) P_I(i), \\ &\sum_{i=1}^{\infty} e\big(i, p(i)\big) P_I(i) \leqslant 1 - r, \end{aligned} \tag{18}$$

where r is a specified data transmission rate, and e is the data error rate as a function of interference and transmitter power. (This elentary formulation generously assumes the receiver interference at each time instant is known to the transmitter.)

In [19] it was shown for the analogous continuous-time problem that if the error rate e is known then the optimal power management function can be found, *independent of the interference distribution*, by finding a stationary point of the associated Lagrangian functional [3,20]

$$F = p - \lambda(1 - e), \tag{19}$$

for some constant (Lagrange multiplier) $\lambda > 0$. The stationary point satisfies

$$0 = \frac{\partial F}{\partial p} = 1 + \lambda \frac{\partial e}{\partial p},$$

or, more simply,

$$\frac{\partial e}{\partial p} = \text{constant}.$$

In the preceding our examples assumed $e = 1/(1+\gamma)$, per equation (5). This may be expressed more generally as

$$e = \frac{b - a}{\gamma + b}, \quad 0 < a < b.$$

The stationary point solution corresponding to this BER is easily found to be

$$p(i) = \sqrt{\lambda(b - a)i} - bi.$$

This provides a fairly general formulation suitable to binary signalling in fading environments (recall the discussion in section 2). For non-fading environments, a formula such as

$$e = a \exp\{-b\gamma\}$$

is preferred (typically with a and b near 1), yielding the optimal solution

$$p(i) = -\frac{i}{b} \ln\left(\frac{i}{ab\lambda}\right).$$

Perhaps most important for the sake of implementation, solutions based on the formulation above were shown in [19] to lend themselves well to a dynamic algorithm. The form of the solution p can usually be found analytically, and then the algorithm need only calculate and update the Lagrange constant as it learns more about its operating (interference) environment. Basic simulations using this technique showed that the use of power management can lead to increased network capacity and stability in addition to reduced energy consumption at the mobile nodes.

Notice that the QOS requirement in the above power management problem formulation is a transmission rate constraint given by (18). However, it is of course true that the target transmission rate r is not the only measure which might form a quality constraint, and it is probably not the most useful one when the data being generated for transmission arrive randomly. In such cases it is more meaningful to consider QOS measures such as average delay (as in [18]) or, in the case of finite (i.e., real) data buffers, blocking or buffer overflow probabilities (also known as loss probabilities or loss rates). Suffice it to say that in systems which are severely delay-constrained, the above form of power management may not be a viable option; in other words, conserving energy may in some cases take a back seat to meeting timing requirements.

When a constant-SIR scheme is used, power levels tend to track one another; in response to higher power from transmitter two, and hence higher interference, transmitter one will increase its power. This produces greater interference at transmitter two's receiver, causing transmitter two to increase its power, and the cycle continues. This behavior, a consequence of equation (4), is well known and the stability consequences well understood (see, e.g., [7,8,10,22,23] and references therein). However, a non-constant-SIR, power-managed system can behave quite differently. Depending on the particular parameters, simulations of power-managed systems similar to those in [19] show that transmitters using power management sometimes track one another, sometimes alternate, and often do a little of both. As opposed to CSPC as expressed in (4), the dynamics are highly nonlinear.

Given this, it is certainly of interest to know if there is a way that one might design a power management scheme to explicitly encourage alternating power levels. To do this it seems necessary to add at least one dimension to the space over which power levels are determined. Below we specifically consider selecting transmitter power as a function of interference *and backlog*, i.e., the number of data elements queued and waiting for transmission. Then the natural QOS constraint becomes an upper bound on the buffer overflow probability (data loss rate), rather than the data transmission rate.

5. Backlog-sensitive power management

Assume that data arrive to the transmitter according to an i.i.d. Bernoulli process, with the probability of an arrival at any instant being $a \in (0,1]$. Consider the problem of selecting transmitter power as a function of queue length in order to minimize energy consumption, subject to ensuring a buffer overflow probability less than or equal to some constant $\beta \in (0,1]$. More precisely, if we assume interference is fixed then we may write

$$\min \sum_{q=1}^{b} p_q \pi_q$$

subject to

$$\pi_b(1-r_b) \leqslant \beta, \qquad \sum_{q=0}^{b} \pi_q = 1.$$

Here b is buffer size, p_q is the transmitter power to be used when the queue length is q, $\{\pi_q\colon q=0,\dots,b\}$ is the steady state distribution of the queue, and r_q is the transmission rate (or departure rate) when the queue length is q. We find by standard computations for the discrete-time birth-death Markov chain associated with the queue (see, e.g., [11]) that

$$\pi_q = \pi_b\left(\frac{a}{1-a}\right)^{q-b} \frac{r_{q+1}\cdots r_b}{(1-r_q)\cdots(1-r_{b-1})}, \quad q=0,\dots,b-1.$$

Of course, to solve we need a relation between r_q and p_q; let us use

$$r_0 = 0 \quad \text{and} \quad r_q = \frac{p_q}{p_q+1}, \quad q=1,\dots,b,$$

noting that any r_q of the form

$$\frac{p_q}{p_q+i}$$

is consistent with (5), taking $e = e_q = 1 - r_q$ and $\gamma = p_q/i$ (recall that we are presently assuming the interference i is fixed). It follows that $p_q = r_q/(1-r_q)$. Omitting details, the objective becomes

$$\min \frac{a^b}{(1-a)^b}\frac{r_b}{(1-r_b)^2} + \sum_{q=1}^{b-1} \frac{r_{q+1}\cdots r_b}{(1-r_q)\cdots(1-r_b)}\frac{a^q}{(1-a)^q}\frac{r_q}{1-r_q}$$

and the constraints become

$$\frac{1}{1-r_b}\frac{a^b}{(1-a)^b} + \sum_{q=0}^{b-1} \frac{r_{q+1}\cdots r_b}{(1-r_q)\cdots(1-r_b)}\frac{a^q}{(1-a)^q} = \beta^{-1}\left(\frac{a}{1-a}\right)^b.$$

Defining $\Lambda = a/(1-a)$, $\mu_q = r_q/(1-r_q)$ and

$$\Phi_q = \begin{cases} \dfrac{r_{q+1}\cdots r_b}{(1-r_q)\cdots(1-r_b)}, & q=0,\dots,b-1, \\ \dfrac{1}{1-r_b}, & q=b, \end{cases}$$

this can be written more concisely as

$$\min \sum_{q=1}^{b} \Phi_q \Lambda^q \mu_q \tag{20}$$

subject to

$$\sum_{q=0}^{b} \Phi_q \Lambda^q = \frac{\Lambda^b}{\beta}. \tag{21}$$

The system (20), (21) can be partially solved using Lagrange' method. Then numerical methods can be used to find values for the r_q, to desired precision.

Example 4. In the case $b=2$ we obtain

$$r_1 = \frac{(\Lambda+1)\lambda - 1}{(\Lambda+1)\lambda + 1}$$

and

$$r_2 = \frac{\Lambda^2(1-\beta)(1-r_1)}{\beta(\Lambda + r_1)},$$

with a rather more complicated expression (omitted) for the Lagrange constant λ in terms of r_1, r_2, and Λ.

Next we will explain how the above development can be used to integrate backlog-sensitivity into an interference-based power management system such as the one discussed in the preceding section. However, a cautionary note is in order: The assumption that the queue is Markovian is certain to be invalid in all but the most contrived of implementations. Hence the rates r_q should be expected to be less than ideal, and future research may be well-directed to alternative schemes for selecting these rates.

We will use the interference-based dynamic power management algorithm (DPMA) found in [19] as a basis. The DPMA can be described by the following sequence of events. TX and RX denote activity at the transmitter and receiver, respectively, and $F = (F_1,\dots,F_n)$ is a (possibly time-varying) partition of the range of received interference. We assume a constant partition and integer-valued interference here to simplify the algorithm.

0. TX: Initialize: $F = 0$, $\lambda = 1$, $t = 0$.
1. RX: Record or estimate interference i_t (or related information) at receiver and signal it back to transmitter.
2. TX: Update t and the interference probability mass function estimate (pmf) based on latest received (time t') information signalled from receiver: increment t, increment $F_{i_{t'}}$, set pmf $= t^{-1}F$.
3. TX: Revise λ based on interference pmf (detailed calculation omitted).[3]
4. TX: Set new power level and transmit: $p = \max\{0, \sqrt{\lambda i_{t'}} - i_{t'}\}$ for a binary fading environment, $p = -i_{t'} \times \ln(i_{t'}/\lambda)$ for non-fading.
5. Continue at step 1.

Steps 1–4 are executed at each time instant, and if the network is stable and stationary (loosely, if transmitters' target rates are achievable and constant over the time interval of interest) then F will gradually reflect the interference distribution encountered on the channel, and the rate of energy consumption will approach a minimum (approximately so except under the ideal conditions of the model, of course).

Because of the way the new transmitter powers are calculated, it is fairly straightforward to integrate backlog-sensitivity into the DPMA. Instead of using a single constant λ to be updated at each time instant (step 3), a vector of b constants λ_q, $q = 1, \ldots, b$, is used. Likewise, a b-vector of rates r_q replaces the single target rate r. Rate r_q enters into the calculation of λ_q. These r_q are precisely the values r_q whose calculation was discussed above. The r_q do not need to be updated regularly; rather, they are simply calculated off-line, once, at the start of operation. As a result of the expanded state space, the transmitter can now make power level decisions based not only on interference, but also on backlog. We refer to the corresponding algorithm as "modified DPMA". To examine the behavior of the various forms of power control considered thus far, we turn to simulation.

6. Network simulation results

We used computer simulation to examine the effectiveness of CSPC, interference-based power management (DPMA), and interference-and-backlog-based power management (modified DPMA), to explore the dynamics of transmitter interaction, and see the extent to which TDMA-like behavior is induced in the DPMA systems. Some operational details concerning the simulator are included in the appendix.

In figure 2 is shown a randomly selected portion of transmitter power sample paths under CSPC. Two transmitters are operating with achievable rates $r_1 = r_2 = 0.45$, so the network is power-stable. The arrival probabilities are $a = 0.4$, and each transmitter has a data buffer of size 10. Receiver noise powers are constant ($\xi_1 = \xi_2 = 1$), the basic binary fading model is employed, and the gain matrix is highly interfering:

$$G = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

As expected, the two transmitters' powers track one another, each occasionally going to zero when its input buffer empties. The system is power-stable (see section 2); hence when both are active their powers approach the steady-state value $p = r_j/(1-2r_j) = 9/2$, which satisfies $r_j = p/(p+i)$ for interference $i = p + \xi_j$.

Now consider figure 3, which shows a modified DPMA system under identical circumstances. (Figure 3 also shows the queue backlogs, labeled "q1" and "q2".) Notice the dramatically different behavior of the power sample paths. It

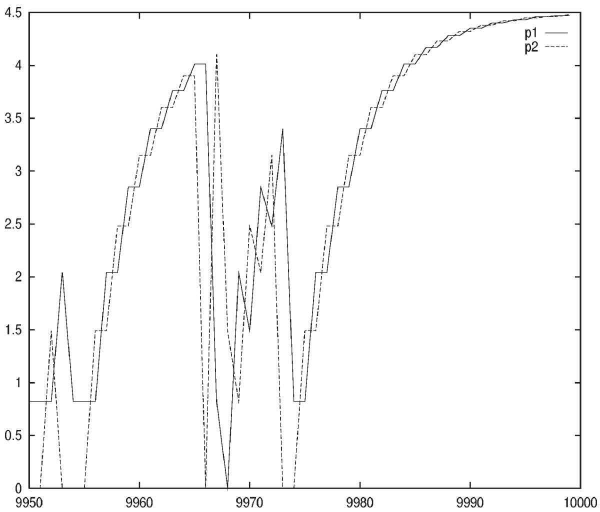

Figure 2. Transmitter powers tend to track each other under CSPC. The two shown here approach the steady-state power 4.5 when both are active.

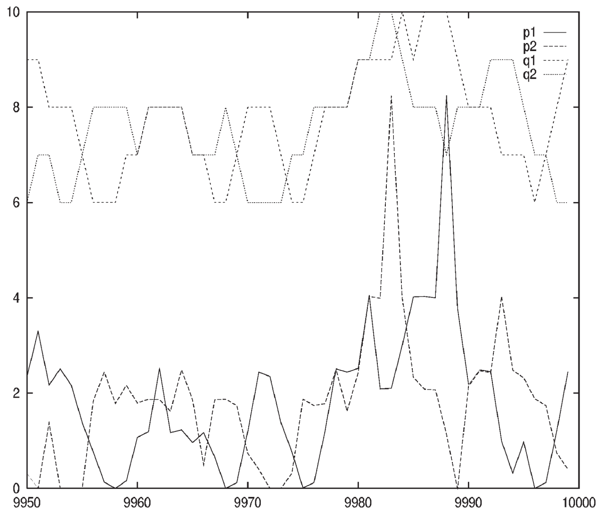

Figure 3. Transmitter powers (and queue lengths) under modified DPMA. Power levels tend to alternate.

[3] An explicit algorithm for the calculation of the Lagrange multiplier λ can be found in [19].

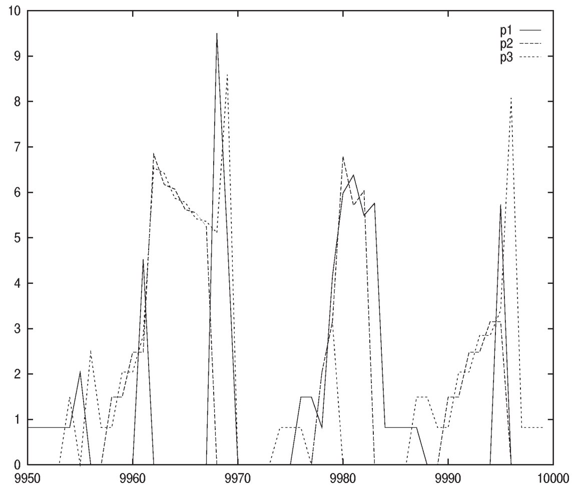

Figure 4. Transmitter powers under CSPC, $N = 3$.

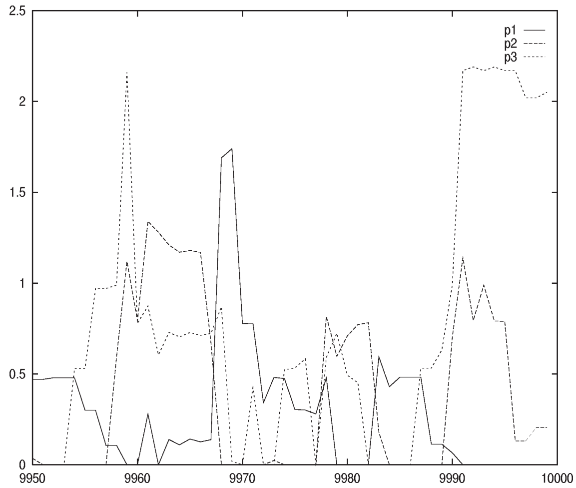

Figure 5. Transmitter powers under modified DPMA, $N = 3$.

is evident that the transmitter powers are alternating, showing at least a partial induced time division effect. The alternating is quite typical, and rather understated relative to many circumstances which produce dramatic on–off behavior. Notice also that transmitter powers "voluntarily" drop to zero even with a non-zero backlog.

It is worth noting again that a power-managed system will in general require larger buffers to achieve the same blocking probabilities as a constant-SIR system, since queue lengths and delays tend to fluctuate more when energy conservation is made a high priority. This is the case in our simulation example above, as the average queue length and blocking probabilities are lower for the CSPC system. For applications which require a compromise of energy savings and tight timing, or for systems which have minimal storage, the power management algorithms covered here can easily be tuned to a compromise setting.

The original DPMA system (not shown) yields transmitter power paths that lie somewhere between the modified DPMA and CSPC, in terms of the degree of induced TDMA effect. Notably, though, the DPMA tends to outperform the two alternatives by operating at lower power for similar rates and loss probabilities. This makes sense in light of the fact that only DPMA makes energy consumption an exclusive priority.

Shown in figures 4 and 5 are roughly comparable samples for $N = 3$ transmitters. Again, the alternating behavior of the modified DPMA scheme is evident relative to the power level tracking which occurs under CSPC for transmitters whose queues are non-empty.

The simulations shown in figures 2–5 all use the binary fading environment model with BERs given by $1/(1 + \gamma)$. One observes the same behavior under a non-fading model, such as with BER $\exp\{-\gamma\}$: transmitters track each other closely when using CSPC, and tend to alternate when using any form of DPMA. Naturally decreasing the off-diagonal gains in the matrix G decreases the interaction between DPMA transmitters.

The simulation used for these tests allows one to vary data arrival rate, arrival and departure batch size, target transmission rates, buffer size, round-trip communication delay, noise distribution and persistence, receiver signal processing gain, maximum power, and many other parameters. Needless to say, it is difficult to present a comprehensive view of such a large model space. While our experimentation has produced no circumstances which contradict the general impressions described above, a few additional observations are in order, which we now summarize.

First, networks with a greater number of mutually interfering transmitters – we have studied as many as ten simultaneously operational nodes – do not exhibit behavior fundamentally different from that observed above. Similarly, while parameters of the ambient receiver noise (distribution and sample path behavior) can affect the operating point of the system, we have observed no evidence that the inter-node dynamics are significantly altered. Finally, simulations of networks in which some of the nodes use CSPC and others use some form of DPMA show that much greater benefit is achieved when instead all users use power management. For example, a typical simulation of $N = 4$ transmitters using the same node parameters as above and gain matrix

$$G = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 1 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix}$$

yielded the results shown in table 1. The upper half of the table shows results for networks in which all four transmitters use the same form of power control (homogeneous); the lower half shows the corresponding results when two CSPC and two DPMA transmitters operate in the same environment. The rows are (1) the percent of arriving data units blocked, (2) average delay before successful transmission, and (3) average transmitter power. In short, the table shows that both types of nodes benefit from the presence of other power-managed transmitters, and the difference is very substantial for nodes using CSPC.

Table 1
Simulation of a four node network, first with all nodes using the same form of power control (top), then with two using DPMA and two using CSPC. Observe that all nodes, especially CSPC nodes, perform better when DPMA nodes are present.

Homog.	Fading		Non-fading	
Parameter	DPMA	CSPC	DPMA	CSPC
blocked pct.	6.7	4.9	5.4	1.4
avg. delay	15.3	13.2	13.1	9.18
avg. power	7.841	599.8	1.818	2.741
Heterog.	Fading		Non-fading	
Parameter	DPMA	CSPC	DPMA	CSPC
blocked pct.	9.3	1.3	4.6	1.8
avg. delay	16.2	8.93	13.0	9.68
avg. power	14.88	17.90	1.908	2.016

7. Conclusion

Our primary aim has been to demonstrate that it is possible to achieve some of the benefits of time division in a network which does not use an explicit time-division frame structure. The modified, backlog-sensitive dynamic power management algorithm shows promise as a mechanism for inducing time-division-like effects. However, the current edition is typically outperformed by unmodified DPMA. Accordingly, one task calling for further investigation is the development of a *dynamic, adaptive* backlog-sensitive algorithm, ideally one that attempts to minimize a user-configurable weighted sum of energy consumption and buffer overflow probability.

Both forms of DPMA induce a form of time division, and as a result do well compared with constant-SIR power control, with the usual tradeoffs (blocking and delay versus energy consumption and stability) duly noted. Network-wide improvements are observed, particularly at nodes which use CSPC, when "cooperative" power management nodes replace "competitive" CSPC nodes.

Finally, an additional area of interest for further research is the issue of mobility. The models above assume all link gains are fixed. It will be interesting to see the effects of mobile nodes, and time-varying gains, on the ability of nodes to capitalize on induced time division.

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Appendix: Some computer simulation details

The simulation, written in C on a GNU/Linux-Intel platform and ported to Digital Unix-Alpha, compiled using the GNU C compiler gcc, runs a 10,000 time step simulation for one or four transmitters on either platform in about two to four seconds (all transmitters using CSPC) or seven to 115 seconds (all using DPMA). This gives a relative (but certainly not absolute) order-of-magnitude estimate of the processing load which can be expected for a mobile device using adaptive power management, though no effort has been made to optimize the DPMA code, and much of the computing time is due to the simulation's data management overhead.

In contrast to the simulator used in [17,19], the one used here incorporated the following two important features: (1) non-zero round-trip communication delay, so information gathered at the receiver arrives after some delay at the transmitter (for the simulations discussed above we used a sliding window protocol with unit time delay); (2) simultaneous updates, so all users update their power levels at each time instant, as opposed to round-robin updating. Of course, both changes lead to more realistic (and less optimistic) results. The performance of the highly nonlinear power management systems was far more susceptible to these changes than the linear CSPC system. Nonetheless, results are encouraging; for a wide range of parameters, the DPMA system outperforms CSPC.

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